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The Dynamic Behaviour of Symmetrical Laminated Nano-composite Containing Equal Numbers of Glass and Carbon Fibre Layers

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Fibre-reinforced polymer composite has many uses in structural components that required high strength, stiffness, and damping capacity. Cross and quasi-laminated epoxy composites with and without nano Al_2O_3 were used in this investigation to determine flexural modulus, natural frequency, damping ratio, and mode shapes by using analytical, experimental, and numerical (ANSYS) methods. It was demonstrated that adding 2 % nano Al_2O_3 improved the flexural modulus and the damping ratio while decreased the natural frequency. Cross number 2 and quasi number 2 had the highest natural frequency for cross and quasi laminate groups which are equal to 23.5 Hz and 20.25 Hz experimentally, respectively. On the other hand, the higher damping ratio was achieved for cross number 1 with nano Al_2O_3 and quasi number 2 with nano Al_2O_3 for both cross and quasi laminates, which are equal to 0.707 % and 0.693 %, respectively. The flexural modulus and damping ratio are inversely related to each other. However, the novelty in this article is that by adding two glass plies at the outer surface of quasi group laminate the flexural modulus, natural frequency, and damping ratio are increased simultaneously, as in the configurations quasi number 2 and quasi number 2 with nano Al_2O_3 in comparison with quasi number 1 and quasi number 1 with nano Al_2O_3.
Key words: cross laminate, quasi laminate, natural frequency, damping ratio, nanoAl_2O_3

Highlights
- *Adding 2 % nano Al_2O_3 decreases natural frequency and increases damping capacity and flexural modulus.*
- *Adding two glass plies at the outer surface of all laminated composite increases flexural modulus and natural frequency, but the damping ratio is also increased jointly in the quasi-laminated group.*
- *Comparison between analytical, experimental, and numerical natural frequency for eight configurations of laminated composites.*

0 INTRODUCTION

Multifunctional fibre-reinforced laminated polymer composites have been utilized in recent decades in structural applications, including automobiles, shafts, aircraft, bicycle frames, and tennis rackets, due to their superior properties, such as low density (low weight), high strength, high stiffness, and high damping capacity. The vibration energy dissipation of fibre-reinforced polymer composite (FRPC) can be achieved via increasing its viscoelastic behaviour [1]. FRPC damping capacity can be enhanced by increasing the numbers of interfacial regions, either by adding fibres or nano-fillers to its viscoelastic matrix, which then causes the vibrating energy to be dissipated by friction between the matrix and reinforcement. Hybridization is one of the means to increase the damping capacity of FRPC by adding high elongation to low elongation fibres [2] and [3].

Natural frequency and damping ratio represent the dynamic behaviour of materials. Glass and carbon fibres are widely use in structural applications. Researchers [4] to [8] have illustrated that the carbon fibre has a higher natural frequency than glass fibre due to the high bending modulus value of carbon fibre in a polymer matrix. In contrast, glass fibre has a higher damping ratio than carbon fibre in a polymer matrix, as shown in Table 1. The mechanical and dynamic properties of materials are related to each other. Zhang et al. [9] and Swolfs et al. [10] illustrated that a laminated fibre composite with equal numbers of glass and carbon fibres gives the best flexural strength and modulus. The high flexural modulus and low strain to failure carbon fabric is added to the low flexural modulus and high strain to failure glass fabric in order to benefit from the advantages of each ingredient and exclude the disadvantages of them. The natural frequency is related to the flexural modulus of the materials, increased with increasing flexural modulus, while the damping ratio is inversely related to the bending modulus. In order to increase interfacial boundary regions, nano-filler is added to the epoxy matrix of interply (G+C) hybrid fibre-reinforced composite, so that more friction will occur when it vibrates; consequently, the damping ratio is increased.

Zhang et al. [11] investigated the tensile properties and damping characteristics in both

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vacuum and air environments of nine conditions of E-glass/polyurethane laminate composites by changing glass volume fraction (V_f) 50 %, 55 %, 60 % and angle of orientation 0°, 45°, and 90°. They deduced that the best damping capacity was for lower glass (V_f) with a greater angle of orientation, which leads to lower tensile strength. The damping capacity in air is higher compared to that in vacuum for the same laminate. Navaneeth et al. [12] studied the tensile, flexural, and damping properties of woven glass/epoxy laminated composites with three different glass volume fractions: 50 %, 60 %, and 70 %. From collected experimental data, they found that the laminate with 60 % glass (V_f) obtained the best tensile and flexural properties (strength and stiffness), while the laminate with 70 % glass (V_f) obtained the highest natural frequency and lowest damping ratio. Hossein et al. [13] researched the effect of adding 1 %, 2 %, 3 %, 5 %, and 7 % nano clay filler to 12-layer woven glass laminated epoxy composite and concluded that increasing the weight percentage of nano-filler to 5 % increased the natural frequency; after that, it decreased, but the damping ratio increased to 7 %. Khahaba [14] researched the impact of adding 1.5 % nano SiC and 1.5 % nano Al_2O_3 on the damping properties of quasi-isotropic laminates with two stacking sequences [0/±45/90] and [90/±45/0]. They proved that the highest damping capacity is obtained for the second stacking configuration with and without nano glass/epoxy composite because the 90° first layer reduces the stiffness of the composites. Pujar et al. [15] examined the effect of introducing 0.1 %, 0.5 %, and 1 % nano-graphene oxide on the damping properties of glass/epoxy composite by utilizing 0° and 45° fibre orientation and two boundary conditions (cantilever and free). They found that adding nano-GO improves damping capacity, whereas 0.5 % GO gives the highest damping ratio. Increasing the angle of fibre orientation between 0° and 45° leads to decrease natural frequency and increase damping ratio for cantilever boundary condition and vice versa for free condition.

Table 1. *Comparative dynamic behaviour of glass and carbon fibre reinforced polymer composite [4] to [7]*

FRPC	Natural frequency [Hz]	Damping ratio [%]
CFRPC	High	Low
GFRPC	low	High

Researchers [4] to [8], [16] to [19] investigated the mechanical and dynamic behaviour of hybrid glass/carbon laminated composite. Murrugan et al. [4] concluded that the presence of carbon fibre in the middle of the laminated composite H1[GCCG] increases tensile strength, tensile modulus, and damping ratio, while the presence of carbon fibre in the outer surface of the laminated composite H2[CGGC] increases flexural strength, flexural modulus, and natural frequency for (50 % carbon: 50 % glass) fibre addition. Suman et al. [5] found that the interply hybridization of equal numbers of glass and carbon fibre in laminated composite affects the dynamic properties. As a result, the arrangement GC1 $[G/C/G/C/G/C/G/C]_s$ has a natural frequency of 46 Hz and a damping ratio of 0.095. In contrast, altering the arrangement by putting the carbon fibre on the external surface for the arrangement CC1$[C/G/C/G/C/G/C/G]_s$ will slightly reduce both the natural frequency and damping ratio to 45 Hz and 0.088, respectively. Pujar et al. [6] investigated the tensile and dynamic properties for laminated composite that is fabricated from 80 % glass and 20 % carbon. They found that H3$[G/G/G/G/C]_s$ has maximum tensile strength and modulus because of the presence of one carbon ply in the middle of the laminate. The dynamic properties were accumulated for free FFFF and cantilever CFFF boundary conditions. In the FFFF condition, a higher natural frequency was found in H1$[C/G/G/G/G]_s$ hybrid condition in the presence of one carbon fibre at the outer surface of the laminate. On the other hand, higher damping ratio was found in H3 hybrid condition in the presence of one carbon fibre in the middle of the laminate. In the CFFF condition, the natural frequency and damping ratio dropped in comparison with FFFF, with the higher natural frequency for first hybrid condition H1, while the higher damping ratio for H2$[G/G/C/G/G]_s$ condition in which one carbon fibre is present after two glass fibres from the outer surface. Adyin et al. [7] predicted the dynamic properties for non-hybrid, interlayer, and intralayer hybrid composites for carbon, glass, and aramid fibre reinforced in an epoxy matrix and taking the angle of orientation, stacking sequence, and number of plies into consideration. They found the best arrangement of fibres for higher dynamic properties by using the Taguchi program. Karthik et al. [16] investigated the damping properties of E-glass chopped mat/woven carbon hybrid with four different volume fractions in epoxy matrix and polyester matrix. They demonstrated that the damping capacity increased with increasing glass volume fraction for both matrices; furthermore, the addition of 5 % carbon fibre gave the best damping behaviour and natural frequency for structural composite. Utomo et al. [17] studied the effect of increasing carbon layers and its position in eight-layer hybrid glass/carbon laminated unsaturated polyester composite and concluded that with increasing numbers of carbon layers near the outer surface of the laminate, the natural frequency increased and damping capacity (ξ) decreased. In contrast, by positioning carbon layers toward the centre of the laminate, the natural frequency decreased while the damping capacity increased. Finally, the natural frequency and flexural stiffness are directly proportional to each other. Singh et al [18] studied the effect of stacking sequence and angle of orientations on the natural frequency and damping ratio of four-layer glass/carbon epoxy laminates and found that the laminate $[0c/90g]_s$ acquired the highest natural frequency, while $[90c/0g]_s$ obtained the lowest one. The highest damping ratio is attained for both laminates $[90g/90c]_s$ and $[90c/90g]_s$, while the laminate $[0c/0g]_s$ obtained the lowest damping ratio. Pingulkar et al. [19] utilized ANSYS software package to evaluate natural frequency and mode shapes of eight-layer glass/carbon hybrid laminated cantilever plates and concluded that important change in natural frequency can be obtained by hybridization, change of angle of orientation, and stacking sequence more than change in volume fraction. Bulut et al. [20] inspected the effect of adding Kevlar fibre into the glass fibre in hybrid laminated epoxy composite with various hybrid ratios through tensile and damping tests. They deduced that H4 (G2K8) had maximum tensile strength and maximum damping capacity; both factors improved, by 124 % and 145 %, respectively, in comparison with glass/epoxy laminate. Alsaadi et al. [21] studied the effect of adding (0.5 %, 1 %, 1.5 %, 2.5 % and 3%) nano-SiO_2 on the tensile, flexural, and damping properties of carbon/Kevlar intraply hybrid epoxy composite. They obtained that the highest tensile modulus and flexural modulus are at 0.5 % nano SiO_2, while the highest tensile strength and flexural strength at 3 % and 1.5 % nano SiO_2, respectively. The flexural modulus and natural frequency are directly related to each other. Therefore, the highest natural frequency at 0.5 % nano-silica which gave 20.5 % improvement, while the damping capacity (damping ratio) decreased by 37 %. Fairlie and Njuguna [22] investigated the impact of stacking sequence and angle of orientation on the tensile and damping capacity of interply hybrid carbon/flax epoxy laminated composite and attained that the outer layer in the laminate is the important layer to control the damping capacity of the laminate. By adding one flax layer at the exterior of the carbon/epoxy laminate, the damping ratio increased by 53.6 % and by adding two flax layers, it increased by 94 %. Alexander et al. [23] studied the effect of boundary conditions, material properties, and laminate thickness on the natural frequency of glass/epoxy and basalt/epoxy laminated composite and reached to the conclusion that the damping capacity of basalt/epoxy composite is higher than that for glass/epoxy composite. Erkliğ, et al. [8] inspected the dynamic behaviour of interply FRPC by using carbon, Kevlar, and glass fibres and determined that $[(0G/90G)3]_s$ and $[(0C/90C)3]_s$ had minimum and maximum natural frequencies, respectively. The laminates $[(0C/90C)3]_s$ and $[(0K/90K)3]_s$ had the minimum and maximum damping capacity, respectively. To increase natural frequency, add higher stiffness fibre at the outer surface, like carbon; however, to increase the damping ratio of the structure, add higher viscoelastic fibre at the surface like Kevlar. Hybrid $[(0C/90C)/(0K/90K)/(0G/90G)]_s$ had the maximum natural frequency compared to other hybrids, and the $[(0/90)3]_s$ fibre orientation had the maximum natural frequency compared to other orientations. Moreover, [24] showed that the large cut-out size decreases the natural frequency, while the small cut-out size had a trivial effect on the natural frequency. Bulut et al. [25] investigated the influence of adding S-glass interply to woven carbon-aramid intraply on the natural frequency and found that higher natural frequency can be obtained by positioning glass in the middle and (C+A) at the outer surface of the laminate, where the configuration $[CA2G2]_s$ had the highest natural frequency. Kröger et al. [26] increased the damping capacity of unidirectional carbon fibre/epoxy laminate by adding vectran, aramid, and cellulose in intraply form. In contrast, tensile, flexural strength, and stiffness were reduced.

Senthamaraikannan and Ramesh [27] decided to increase the damping capacity of woven carbon laminated structure by adding 11 % nano SiO_2 with 9 % micro CTBN rubber to the epoxy matrix, despite the slight drop in tensile and flexural modulus. Bulut et al. [28] investigated the dynamic behaviour of basalt/epoxy laminated composite by adding (0.1, 0.2, and 0.3) nano-graphene and determined that 0.1 % and 0.2 % of nano-pellets increased the natural frequency and damping ratio, but 0.3 % decreased both of them.

According to the above literature review, it is clear that increasing flexural modulus is accompanied by increasing the natural frequency and decreasing the damping ratio. One way to increase flexural modulus is by adding equal numbers of glass and carbon fibre layers to the composite matrix and then rearranging them to obtain a higher damping capacity. Another way to increase flexural modulus and damping capacity is by adding nearly 2 % nano-filler to the matrix of fibre-reinforced polymer composite.

The aim of this investigation is to increase flexural modulus, natural frequency, and damping capacity simultaneously by using a new special arrangement of glass and carbon fibres that are different from the cited studies and by adding 2 % nano Al_2O_3 to the epoxy matrix of G/C hybrid composite. Eight configurations for eight layers of glass/carbon epoxy laminated composite with and without nano Al_2O_3 were fabricated. Analytical, experimental, and numerical (ANSYS) methods were used to evaluate the flexural modulus, natural frequencies, damping ratio, and shape modes, in addition to compare between them.

1 ANALYTICAL WORK

Knowledge of the dynamic properties of FRPC is just as important as that of the static ones. It is suitable to utilize static stiffness to estimate the natural frequency of laminated composite [2]. Classical laminate plate theory (CLPT) was used to evaluate the theoretical flexural modulus and fundamental natural frequency for all laminates (shown in Table 2) by using the mechanical properties for unidirectional glass and carbon fibres with and without nano-Al_2O_3 (shown in Table 3).

The flexural stiffness matrix and theoretical flexural modulus for each laminate can be calculated by using Eqs. (1) and (2) [29].

$$\mathbf{D_{ij}} = \frac{1}{3}\sum_{k=1}^{n}\left[\left(\mathbf{Q_{ij}}\right)\right]_k \left(h_k^3 - h_{k-1}^3\right), \tag{1}$$

$$E_b = \frac{12}{h^3 D_{11}^*}. \tag{2}$$

Table 2. *Stacking sequence G: C configuration of hybrid laminated epoxy composite*

Symbol	Laminates	Stacking sequences	Hybrid ratio G:C
C1	Cross no. 1	$[G0/C90/C0/G90]_s$	4:4
C1WN	Cross no. 1 with nano Al_2O_3	$[G0/C90/C0/G90]_s$	4:4
C2	Cross no. 2	$[G0/G90/C0/C90]_s$	4:4
C2WN	Cross no. 2 with nano Al_2O_3	$[G0/G90/C0/C90]_s$	4:4
Q1	Quasi no. 1	$[G0/C90/C45/G\text{-}45]_s$	4:4
Q1WN	Quasi no. 1 with nano Al_2O_3	$[G0/C90/C45/G\text{-}45]_s$	4:4
Q2	Quasi no. 2	$[G0/G90/C45/C\text{-}45]_s$	4:4
Q2WN	Quasi no. 2 with nano Al_2O_3	$[G0/G90/C45/C\text{-}45]_s$	4:4

Table 4 demonstrates the [**D**] matrix, theoretical flexural modulus E_b, and fundamental natural frequency *wn* for all laminated epoxy composites. The fundamental frequency is the smallest frequency for the structure; for the case of a cantilever beam, *wn* can be calculated from Eq. (3) [30]

$$wn = \frac{1}{2\pi}\left(\frac{1.875}{l}\right)^2 \sqrt{\frac{E_b I_{yy}}{\rho A}} \tag{3}$$

Table 3. *The mechanical properties of unidirectional glass and carbon with and without nano alumina*

Laminates	Density [kg/m³]	E_1 [MPa]	E_2 [MPa]	v_{12}	G_{12} [MPa]
UD glass/epoxy	1658	31802	12804	0.22	4271
UD carbon/epoxy	1484	99438	6273	0.25	4031
UD glass/epoxy Al_2O_3	1883	33507	13344	0.27	4500
UD carbon/epoxy Al_2O_3	1574	105044	6626	0.3	4260

The results in Table 4 show that cross-laminate has higher flexural modulus than quasi-laminate, similar to the results of [7] and [17]. It is apparent that C2WN and Q2WN laminates have maximum flexural modulus E_b in the cross and quasi groups, respectively. In spite of the presence of two glass layers on the exterior of both laminates, it is in contrast with the results of [16] because of the effect of element Q11, Q12, and Q66 in [$\mathbf{Q_{ij}}$] matrix of the 90^oglass layer. By adding 2 % nano Al_2O_3, the natural frequency for all laminates slightly dropped despite the increase in flexural modulus, similar to the results of research [14]. The maximum natural frequency had been obtained for C2 in the cross-group, while in the quasi-group, Q2 had the maximum natural frequency.

2 EXPERIMENTAL

2.1 Materials

The matrix material used in this study is the laminating epoxy resin MGS L285 with hardener H285 in a 100:40 mixing ratio (resin: hardener), for which the density for resin and hardener are equal to 1.18 g/cm³ to 1.23 g/cm³ and 0.94 g/cm³ to 0.97 g/cm³ respectively, while the viscosity for them is equal to 600 mPa·s to 900 mPa·s and 50 mPa·s to 100 mPa·s, respectively. Unidirectional carbon and E-glass fabric were used as fibre reinforcement with weights equal to 300 g/cm² and 330 g/cm², respectively. Both the matrix and fibre were supplied by the DOST KIMYA Company, Turkey. Finally, spherical aluminium oxide (Al_2O_3) nano-powder/nano-particle with a size of 48 nm, which was used as a filler reinforcement, was supplied by Nanografi Nanotechnology Company, Turkey.

Table 4. *Flexural stiffness matrix, theoretical flexural modulus E_b, and fundamental natural frequency wn*

Laminates	Flexural stiffness matrix [**D**] [Pa·m³]	Theoretical flexural modulus, E_b [GPa]	Fundamental natural frequency, wn [Hz]
C1	$\begin{bmatrix} 21.2 & 1.56 & 0 \\ 1.56 & 25.6 & 0 \\ 0 & 0 & 2.78 \end{bmatrix}$	31.6	23.254
C1WN	$\begin{bmatrix} 22.46 & 2.01 & 0 \\ 2.01 & 27.05 & 0 \\ 0 & 0 & 2.94 \end{bmatrix}$	33.5	22.75
C2	$\begin{bmatrix} 22.43 & 1.81 & 0 \\ 1.81 & 12.95 & 0 \\ 0 & 0 & 2.83 \end{bmatrix}$	33.3	23.78
C2WN	$\begin{bmatrix} 23.79 & 2.33 & 0 \\ 2.33 & 13.71 & 0 \\ 0 & 0 & 2.98 \end{bmatrix}$	35.1	23.29
Q1	$\begin{bmatrix} 16.21 & 3.21 & 1.65 \\ 3.21 & 27.26 & 1.65 \\ 1.65 & 1.65 & 4.43 \end{bmatrix}$	23.0	19.78
Q1WN	$\begin{bmatrix} 17.24 & 3.74 & 1.75 \\ 3.74 & 28.83 & 1.75 \\ 1.75 & 1.75 & 4.66 \end{bmatrix}$	24.4	19.42
Q2	$\begin{bmatrix} 17.7 & 3.62 & 1.46 \\ 3.62 & 14.07 & 1.46 \\ 1.46 & 1.46 & 4.64 \end{bmatrix}$	24.75	20.52
Q2WN	$\begin{bmatrix} 18.8 & 4.23 & 1.55 \\ 4.23 & 14.91 & 1.55 \\ 1.55 & 1.55 & 4.88 \end{bmatrix}$	26.0	20.05

2.2 Laminates Manufacturing

In this investigation, four hybrid glass/carbon laminates were used without nano-particles and the other four with it. The 2 % nano Al_2O_3 were added to epoxy matrix by using dual mixing method, which includes both ultrasonic vibration and magnetic stirring mixing simultaneously. After that, the bubbles produced are removed by degassing the suspension mixture. The glass and carbon fibre fabrics were cut into the required angles and dimensions, and then arranged according to the design stacking sequence in order to start the vacuum assisted resin infusion method (VARIM) by drawing the epoxy mixture into the closed system of arranged fibres and VARIM accessories **[31]**. A flow chart for the purpose of each fabrication stages is shown in Fig. 1.

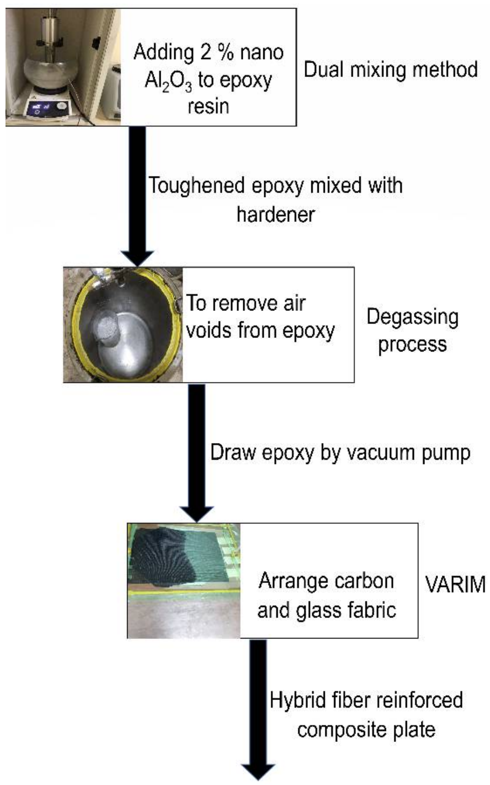


Fig. 1. *Stages of laminated composite fabrication*

2.3 Free Vibration Test

The dynamic properties of FRPC beam with dimensions are 250 mm length × 25 mm width and 2 mm thickness; they were measured by using the experimental set up shown in Fig. 2, which consist of the following apparatuses. Firstly, an impact hammer transducer was used to excite an impulsive force and measure it at the midpoint of the beam (Brüel & Kjær, type 8206). Secondly, a piezoelectric accelerometer was used to measure the vibration response of the excited beam (Brüel & Kjær, type 4507 B30515) positioned at the free end of the beam. Finally, the both hammer and accelerometer were connected to the Brüel & Kjær controller modules type 7539A, 5-channels, in order to analyse the collected data by using fast Fourier transform (FFT) Analyzer to obtain the dynamic response of the free vibration test, in terms of time domain and frequency domain.

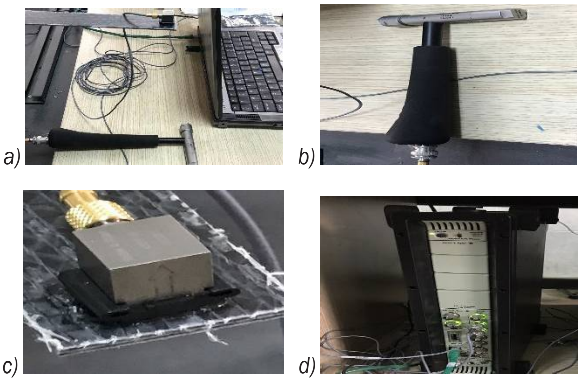


Fig. 2. *Free vibration test set up and its parts; a) free vibration set up, b) impact hammer, c) accelerometer sensor, and d) blank module*

Due to FRPC beam free vibration, the dynamic behaviour of the glass/carbon epoxy laminated composite can be found in terms of time domain (displacement-time envelope) and frequency domain (acceleration-frequency envelope). The original aspect of this paper is that the frequency response is neither represented in terms of input impulsive force with respect to frequency nor within output acceleration with respect to frequency; the new representation was in terms of output over input (acceleration/impulsive force) in the y-axis against the frequency in the *x*-axis, as shown in Fig. 3. The red curser position in Fig. 3 represents the resonant frequency, for the range of frequency is between 0 Hz and30 Hz, as represented by the critical frequency for the laminates and not necessarily the maximum frequency. If the laminate is subjected to force with excited frequency equal to this

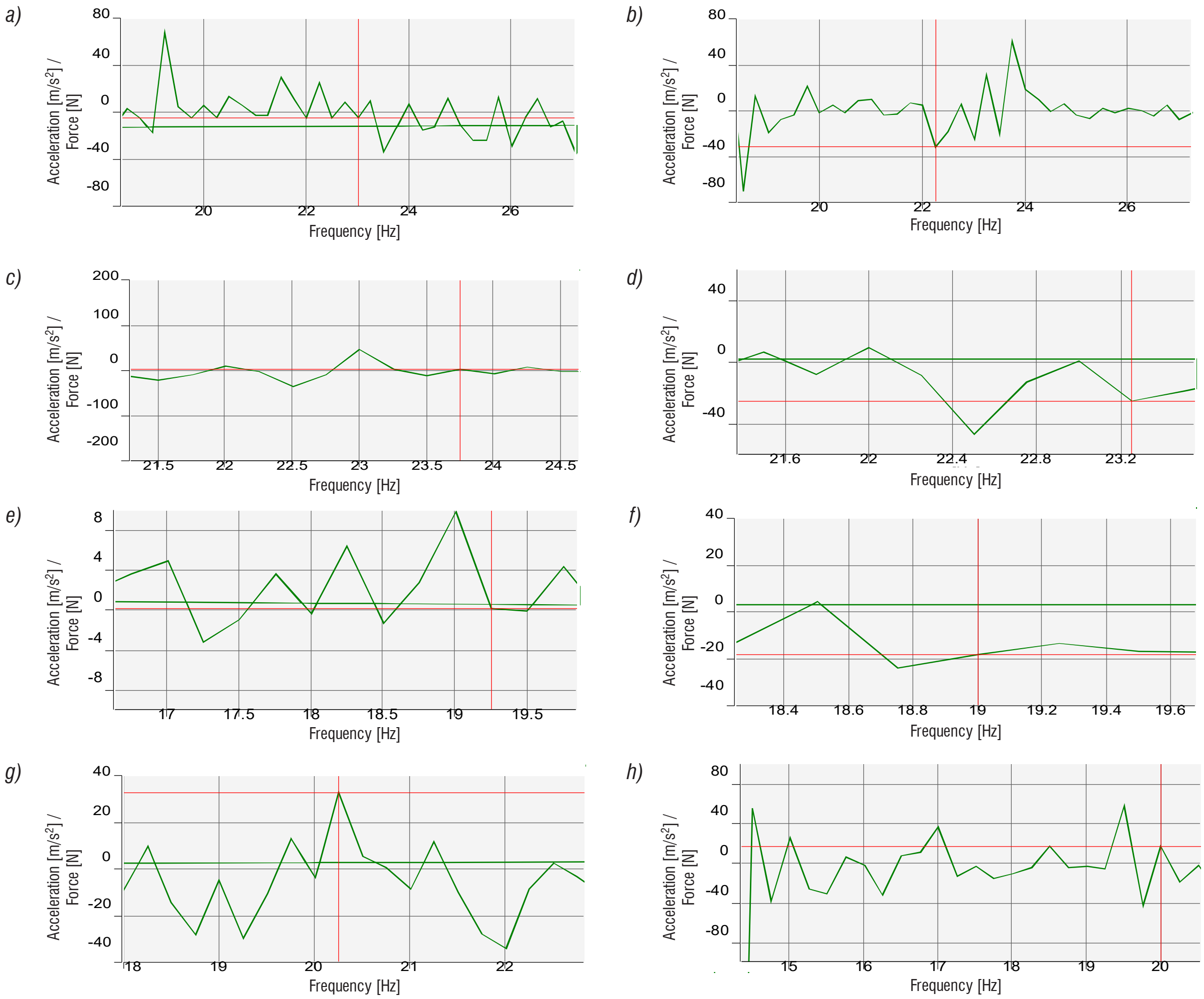


Fig. 3. *Frequency responses of laminated epoxy composites with and without nano Al_2O_3; a) C1, b) C1WN, c) C2, d) C2WN, e) Q1, f) Q1WN, g) Q2, and h) Q2WN*

resonance frequency, then the amplitude of vibration pulse will be magnified and cause failure and presence of crack in the same laminate. The dynamic responses of all laminates are illustrated in Fig. 4 in terms of time domain. The number of peaks for cross- and quasi-group vibration response to a 400 ms time are 5 and 4 peaks respectively; this is due to the increase of natural frequency of cross group in comparison with quasi group as shown in Tables 4 and 5, similar to the results of studies [7] and [18]. The maximum natural frequency for the laminate C2 is equal to 23.5 Hz in cross group. On the other hand, the maximum one in the quasi group is for the laminate Q2 and equal to 20.25 Hz as shown in Fig. 5 because the flexural modulus of the arrangement of GGCCCCGG organized by two glass plies at the outer surface is higher than that of the arrangement GCCGGCCG as shown in Table 5 which is different from the results of studies [6] and [17]. The element Q11, Q12, and Q66 of [$\mathbf{Q_{ij}}$] matrix for 90^o glass fibre is higher than that for 90º carbon. Therefore, the presence of 90^o glass as a second layer in the laminate leads to increase the first element of [**D**] matrix and then the flexural modulus. By adding 2% nano Al_2O_3 to all laminated composites, the amplitude of the vibration response decreased in comparison with the original state as shown in Fig. 4. This is due to the increase of the damping ratio for the nano particle addition case in comparison with the non-addition case, similar to the results of researches [11], [13] to [17], [21], [27], and [28] as shown in Table 5 and Fig. 6. This is occurred because of the increase of the interfacial regions in composite leading to more

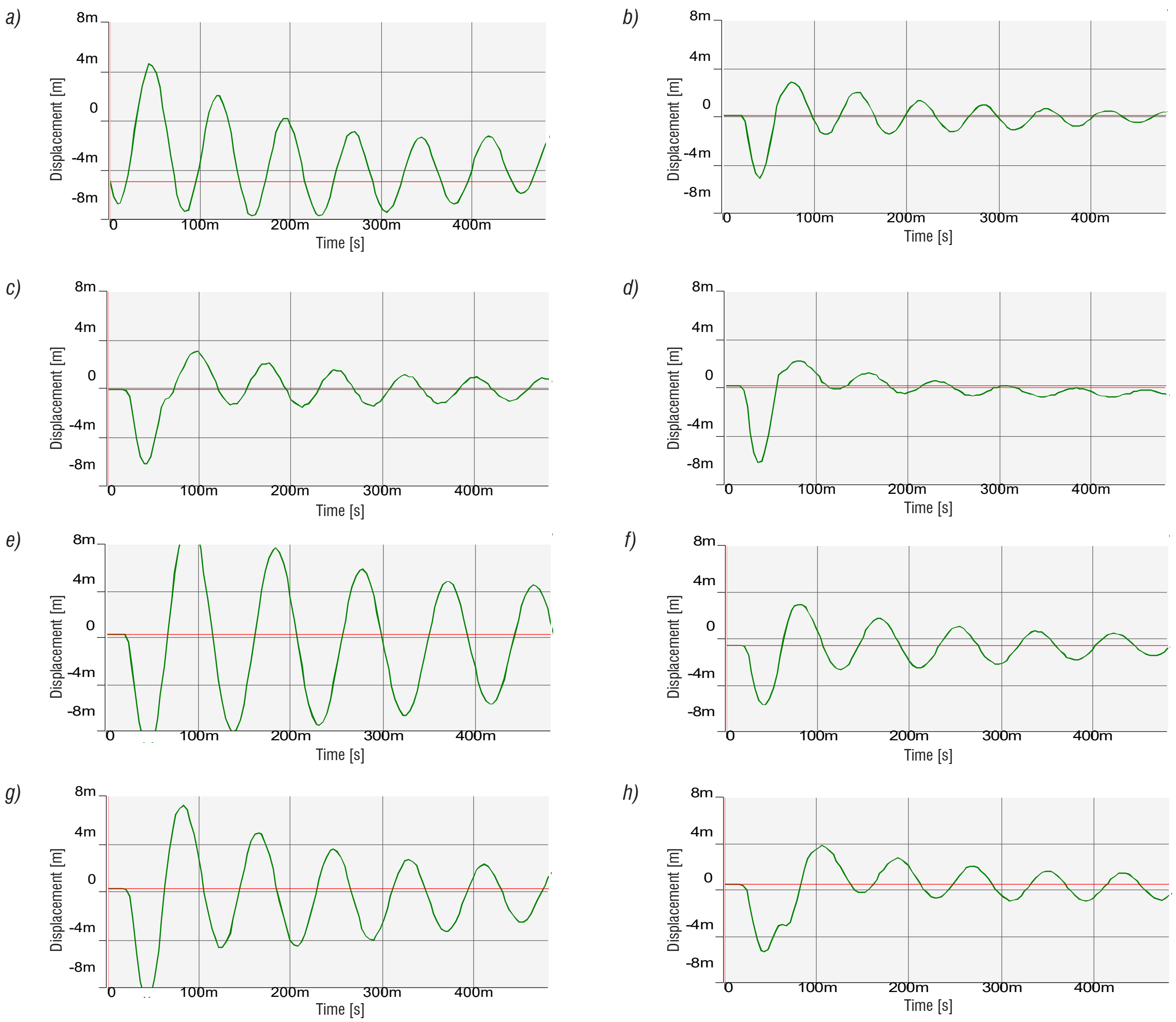


Fig. 4. *Vibration responses of laminated epoxy composites with and without nano Al_2O_3: a) C1, b) C1WN, c) C2, d) C2WN, e) Q1, f) Q1WN, g) Q2, and h) Q2WN*

energy dissipation by friction (heat) [2] and [3]. Despite the amplitude of the dynamic response for C2, C2WN, Q2, and Q2WN being lower than the amplitude for C1, C1WN, Q1 and Q1WN, respectively, as shown in Fig. 4, but the damping ratio of C2 is lower than the damping ratio of C1 by 24.2 %, and the damping ratio of C2WN is lower than the damping ratio of C1WN by 8 %. Its behaviour is due to the fact that the ratio of the maximum peak amplitude to the minimum peak amplitude is higher for C1 and C1WN than that for C2 and C2WN, respectively as shown in Fig. 4. The reverse situation for quasi group, where the damping ratio for Q2 is higher than damping ratio of Q1 by 5.62 % and the damping ratio for Q2WN is higher than the damping ratio of Q1WN by 6.64 % is due to the fact that the ratio of the maximum peak amplitude to the minimum peak amplitude is higher for Q2 and Q2WN than Q1 and Q1WN, respectively, as shown in Fig. 4. The stacking sequence and angle of orientation have a major effect on the value of the bending modulus, natural frequency, and damping ratio. For structural material system, it is necessary to make a balance between these properties alongside its strength. The configuration Q2 and Q2WN $[G0/G90/C45/C\text{-}45]_s$ is specialized by increasing flexural modulus, natural frequency, and damping ratio simultaneously, in comparison with Q1 and Q1WN $[G0/C90/C45/G\text{-}45]_s$, respectively, as shown in Tables 4 and 5.

Table 5. *The experimental free vibration results (natural frequency & damping ratio)*

Laminates	Natural frequency [Hz]	Damping ratio [%]
C1	22.75	0.629
C1WN	22.25	0.707
C2	23.5	0.477
C2WN	23	0.651
Q1	19	0.487
Q1WN	18.5	0.647
Q2	20.25	0.516
Q2WN	19.5	0.693

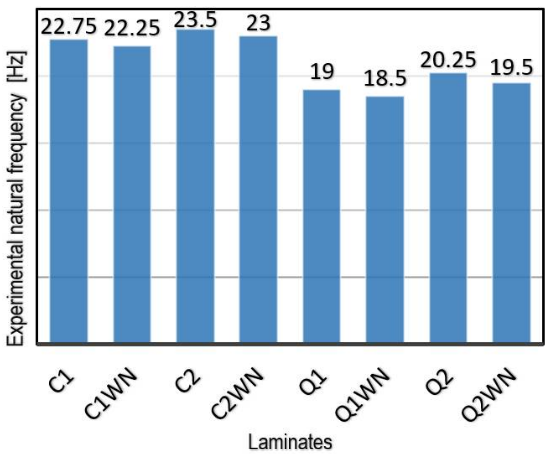


Fig. 5. *Natural frequencies of all laminates with and without nano Al_2O_3*

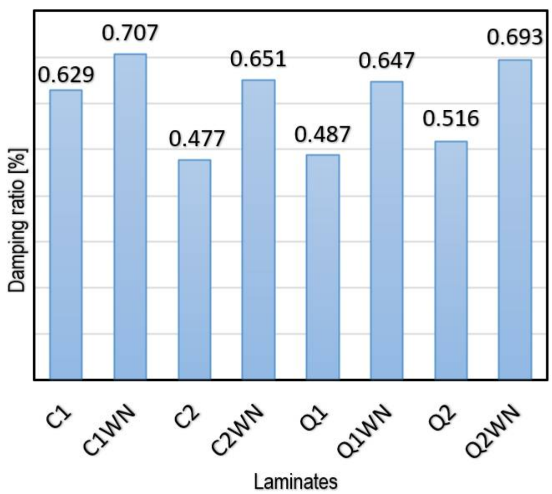


Fig. 6. *Damping ratios of all laminates with and without nano Al_2O_3*

3 NUMERICAL WORKS

Ansys workbench simulation package was used to find the first six natural frequencies and its mode shape and compare it with the analytical and experimental one by using the steps shown in Fig. 7.

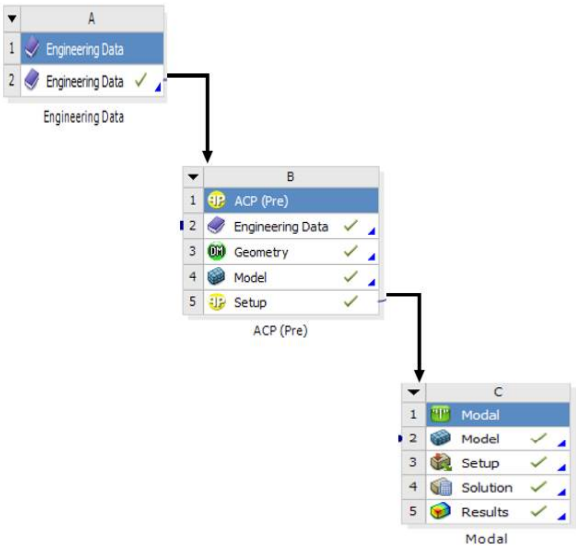
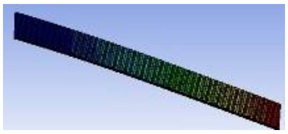
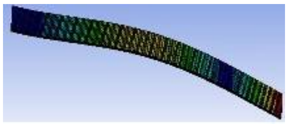
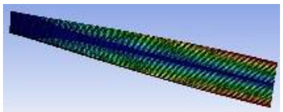
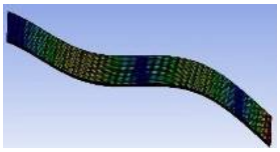
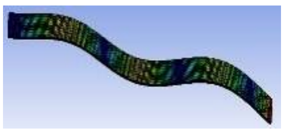


Fig. 7. *Steps of modelling laminates in ANSYS*

Firstly, the mechanical properties for each lamina must be entered to the engineering data. Then the arrangement of fibres in the laminated composite was defined by using Ansys Composite PrepPost ACP (Pre) to create FEM models for different stacking sequences of laminates and finally the model analysis was used to evaluate until the first six natural frequencies with their mode's shapes are given, as shown in Table 6. It is obvious from Table 6 and Fig. 8 that all modes are bending modes, except mode 4 is torsional mode. Moreover, the maximum natural frequency for 1st, 2nd, 3rd, 5th, and 6th modes existed in C2 laminate, while for 4th mode, the maximum natural frequency existed in Q2, where in the laminates C2 and Q2 stacking sequence of G/G/C/C/C/C/G/G was utilized.

Finally, it is found that the analytical, experimental, and numerical natural frequencies are very close to each other, as shown in Fig. 9.

Table 6. *The first six natural frequencies and its mode shapes for all laminates*

No	Mode shape	Natural frequency [Hz]							
		C1	C1WN	C2	C2WN	Q1	Q1WN	Q2	Q2WN
1		23.185	22.752	23.8	23.313	19.821	19.452	20.59	20.138
2		145.16	142.45	148.98	145.94	124.08	121.77	128.9	126.07
3		263.89	258.42	265.8	260.2	218.92	214.86	231.4	225.78
4		309.84	303.71	309.75	303.59	320.66	313.71	326	318.95
5		405.87	398.29	416.45	407.94	347.04	340.58	360.5	352.71
6		793.69	778.86	808.91	791.91	679.17	666.55	705.7	690.54

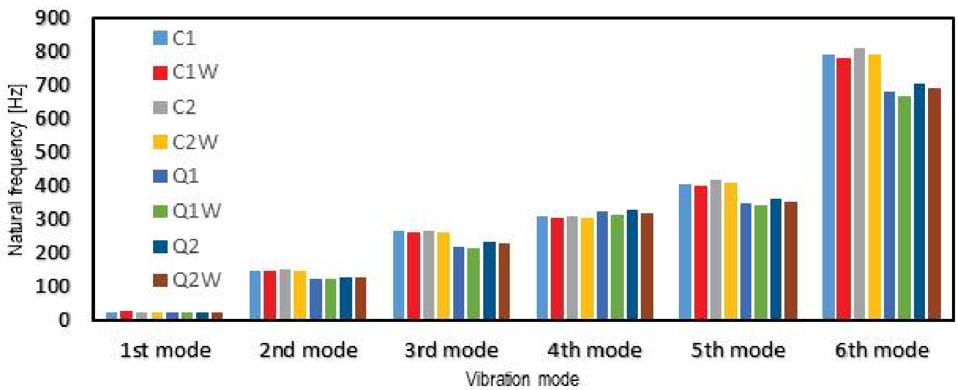


Fig. 8. *Numerical natural frequency for all laminated composites*

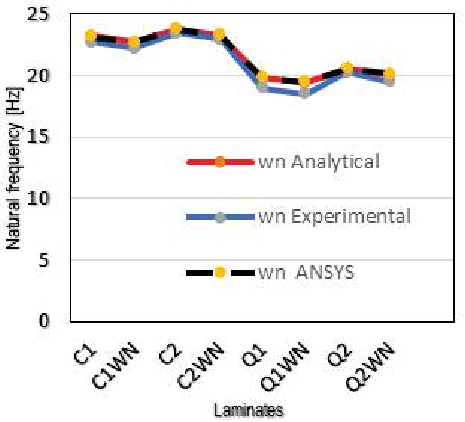


Fig. 9. *Natural frequencies comparison in analytical, experimental, and numerical methods*

4 CONCLUSIONS

Structural components need to have high strength and stiffness along with high damping capacity. In this paper, the following conclusions can be made.

1. The maximum natural frequency for the cross- and quasi-laminate group was obtained for C2 and Q2 laminates.
2. The relationship between the natural frequency and damping ratio is inversely proportional.
3. Both flexural modulus and damping ratio were increased with addition of 2 % nano-Al_2O_3, while the natural frequency was decreased.
4. The maximum damping ratio for the cross- and quasi laminate group was obtained for C1W and Q2W laminates, respectively.
5. The new aspect of this work is in the Q2 and Q2W configurations, which is appropriate to use in industry for structural element parts with higher bending modulus, natural frequency, and damping ratio properties.
6. The analytical, experimental, and numerical (ANSYS) natural frequencies are very close to each other.

5 NOMENCLATURES

$\mathbf{D_{ij}}$	flexural stiffness matrix, [Pa·m³]
$[\mathbf{Q_{ij}}]$	reduced stiffness matrix, [Pa]
h	laminate thickness, [mm]
D_{11}	first element in the flexural compliance matrix [1/(Pa·m³)]
E_b	flexural modulus, [GPa]
wn	fundamental natural frequency, [Hz]
l	beam length, [m]
I_{yy}	moment of inertia, [m⁴]
ρ	density, [kg/m³]
A	cross sectional area, [m²]

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