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RESEARCH ARTICLE

Iterative Kernel Technique to Solve System Fredholm Integral Equation First Kind for Degenerate Kernel

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ABSTRACT

Many problems associated with the engineering technology field can be transformed into Fredholm integral equations of the first kind to achieve problem-solving strategies. In this paper, the iterative kernel technique was reformulated to treat the numerical solution for the system of Fredholm integral equations of the first kind for the degenerate kernel. Three new theorems have been proposed and proved. This technique was programmed via Matlab and achieved a good result

Keywords: Iterative kernel technique, the system of integral equations, degenerate kernel, and Fredholm

INTRODUCTION

Integral equations are useful for representing a variety of scientific phenomena. In the field of applied sciences various phenomena transformed to problems in an integral equation, many integral equations issues were built from numerous examples in various areas of applied sciences such as mathematical physics, engineering, and chemistry [Burova, I. G. and Ryabov, V. M. 2020, Wazwaz M. A. 2011]. Recently, several scientists reformulated various techniques for solving many types of problems of integral equations, several papers have been published with scientific phenomena and paved away to develop several high-accuracy numerical techniques [Hassan, T. I Sulaiman N. A. and Saleh S. 2016, Jose M. G. and Miguel A. H. 2021, Edalatpanah, S. A. and Abdulmaleki, E. 2014, Long G. and Nelakanti G. 2007, Talaat, I. H. 2019].

The system of Fredholm integral equation appears from various phenomena in mathematics, electromagnetic waves, and communication problems. This system is constructed especially from a system of boundary value problem [Ibrahim, H. Attah, F. and Gyegwe, T. 2016, Maxime, B. A. 1971].

On the other hand, this system was an ill post problem so that it cannot be solved essay for all types of kernels, for this cause choosing degenerate kernel type for solving it as a special case. In general, various techniques with sufficient accuracy and efficiency have been utilized before by many researchers to solve problems in integral equations, iterative Scheme is one of these techniques [Abdou M. A. 2002, Hassan T. I. 2019, Jose M. G. and Miguel A. H. 2021, Sahu, P. K. and Saha Ray, S. 2014, Wazwaz 2011].

The primary purpose of this paper is to reformate and apply the iterative kernel technique for solving the system of Fredholm integral equations of the first kind with degenerate kernel type

2- DEFINITIONS

Introducing some definition.

Definition 1: The system of Fredholm integral equations of the first kind is defined as follows

$$h_i(v) = \sum_{j=1}^{q} \beta_{ij} \int_a^b Q_{ij}(v,z)p_j(z)dz \qquad (1)$$

where $m \in N$, $h_i(v)$ and $Q_{ij}(v,z)$ for $i = 1,2,\dots,m$ continuous known functions on $[a,b]$ respectively, $G = \{(v,z): a \le v < z \le b\}$ such that the functions, $p_j(z)$ is unknown [Wazwaz, A.M. 2011].

Definition 2:

if $Q(v,z) = \sum_{y=1}^{n} d_y(v) r_y(z),\ \forall (v,z) \in G$

then $Q(v,z)$ is known as degenerate kernel [Maxime, B. A. 1971, Wazwaz, A.M. 2011].

3- USING ITERATIVE KERNEL TECHNIQUE TO SOLVE EQUATION ONE.

Recently many researchers applied this technique because it has a significant role in solving various problems. Also, different problems in various areas of applied mathematics were solved by this technique. Here driving this technique. For solving this problem. Assume equation (1) has numerical solution as,

$$p_i(v) = \sum_{j=1}^{n} \beta_{ij} p_i^j(v) \text{ for } i = 1,2,\dots,m \qquad (2)$$

The $p_i^j(v)$ defined in [Sulaiman, N. A. and Hassan, T. I. 2008] as,

$$p_i^j(v) = \sum_{j=1}^{q} \int_a^b Q_{ij}(v,z) p_i^{j-1}(z)dz \qquad (3)$$

Then by using equation (3) $p_i^1(v)$ can find by

$$p_i^1(v) = \sum_{j=1}^{q} \int_a^b Q_{ij}(v,z) p_i^0(z)dz \qquad (4)$$

Let $p_i^0(v) = h_i(v)$

$$p_i^1(v) = \sum_{j=1}^{q} \int_a^b Q_{ij}(v,z) h_i(z)dz$$

then $p_i^2(v)$ is calculated as follows

$$p_i^2(v) = \sum_{j=1}^{q} \int_a^b Q_{ij}(v,z) p_i^1(z)dz$$

$$= \sum_{j=1}^{q} \int_a^b Q_{ij}(v,z)[\sum_{r=1}^{q} \int_a^b Q_{ir}(v,k) h_r(k)dk]dz$$

$$= \sum_{j=1}^{q} \int_a^b [\sum_{r=1}^{q} \int_a^b Q_{ij}(v,z)\, Q_{ir}^1(z,k)dz] h_r(k)dk$$

Let $Q_{ir}^1(z,k) = Q_{ir}(z,k)$ then

$$p_i^2(v) = \sum_{j=1}^{q} \int_a^b Q_{ij}^2(v,k) h_j(k)dk \qquad (5)$$

Such that $Q_{ij}^2(v,k)$

$$Q_{ij}^2(v,k) = \sum_{r=1}^{q} \int_a^b Q_{ij}(v,z)\, Q_{ir}^1(z,k)dz \qquad (6)$$

Calculating $p_i^3(v)$ by

$$p_i^3(v) = \sum_{j=1}^{q} \int_a^b Q_{ij}(v,z) p_i^2(z)dz, \qquad (7)$$

for $i = 1,2,\dots,m$ substitute equation (5) into equation (7) then

$$p_i^3(v) = \sum_{j=1}^{q} \int_a^b Q_{ij}(v,z)[\sum_{r=1}^{q} \int_a^b Q_{ir}^2(v,k) h_r(k)dk]dz$$

$$= \sum_{j=1}^{q} \int_a^b [\sum_{r=0}^{q} \int_a^b Q_{ij}(v,z)\, Q_{ir}^2(z,k)dz] h_r(k)dk$$

Such that $Q_{ij}^3(v,k) = \sum_{r=1}^{q} \int_a^b Q_{ij}(v,z)\, Q_{ir}^2(z,k)dz$

Hence

$$p_i^3(v) = \sum_{j=1}^{q} \int_a^b Q_{ij}^3(v,k) h_j(k)dk \qquad (8)$$

Therefore, the numerical solution given by,

$$p_i^n(v) = \sum_{j=1}^{q} \int_a^b Q_{ij}^n(v,k) h_j(k)dk \qquad (9)$$

for $i = 1,2,\dots,m$ and the n^{th} iterative kernels is

$$Q_{ij}^n(v,k) = \sum_{r=1}^{q} \int_a^b Q_{ij}(v,z)\, Q_{ir}^{n-1}(z,k)dz \qquad (10)$$

The sum of $p_i^0(v), p_i^1(v), p_i^2(v), \dots$, where $i = 1,2,\dots,m$ is converge to the numerical solution for the main problem.

4- THEOREMS

Proving several theorems, which relate to the main problem.

Theorem 4.1

Assume that $Q(v,z)$ and $P(v,z)$ be degenerate kernel then $Q(v,z)P(v,z)$ is also degenerate kernel $\forall, (v,z) \in G$.

Proof: Since $Q(v,z)$and $P(v,z)$ are degenerate kernel then

we have
$Q(v,z)=\sum_{i=1}^{n} d_i\,(v) r_i(z)$ and
$P(v,z)=\sum_{j=1}^{m} d_j^*\,(v) r_j^*(z)$ then

$$Q(v,z)P(v,z)=\sum_{i=1}^{n} d_i\,(v) r_i(z) \sum_{j=1}^{m} d_j^*\,(v) r_j^*(z)$$
$$=\sum_{i=1}^{n}\sum_{j=1}^{m} d_i\,(v) r_i(z) d_j^*(v) r_j^*(z)$$
$$=\sum_{i=1}^{n}\sum_{j=1}^{m} d_i\,(v) d_j^*(v) r_i(z) r_j^*(z)$$
$$=\sum_{i=1}^{n}\sum_{j=1}^{m} D_{ij}\,(v) R_{ij}(z)$$

Hence it's the degenerate kernel.

Theorem 4.2
Let $Q(v,z)$ and $P(v,z)$ be two non-identically zeros and continuous functions
$\forall, (v,z)\in G$ then $Q(v,z)P(v,z)$ non-identical zero function [Maxime, B. A. 1971].

Theorem 4.3
Let $Q_{ij}(v,k)$ be non-identically zeros and continuous functions $\forall, (v,z)\in G$ then
$Q_{ij}^1(v,k),\ Q_{ij}^2(v,k),\ Q_{ij}^3(v,k),\dots,Q_{ij}^n(v,k)$ degenerate kernel and non-identically zeros.

Proof: 1) To show that
$Q_{ij}^1(v,k), Q_{ij}^2(v,k),\ Q_{ij}^3(v,k),\dots,\ Q_{ij}^n(v,k)$ are the degenerate kernel, by mathematical induction, for $n=1$ we have
$Q_{ij}^1(v,k)=Q_{ij}(v,k)=\sum_{y=1}^{n} d_y\,(v) r_y(k),\ \forall(v,k)\in G$
then it is holds.
Let $Q_{ij}^n(v,k)$ is degenerate kernel for $n=u$ then we have

$$Q_{ij}^u(v,k)=\sum_{r=1}^{q}\int_a^b Q_{ij}\,(v,z)\,Q_{ir}^{u-1}(z,k)dz=$$
$$\sum_{r=1}^{q}\int_a^b\left[\sum_{s=1}^{n} d_{is}\,(v) r_{js}(z)\sum_{t=1}^{m} d_{it}^{u-1}\,(z) r_{rt}^{u-1}(k)\right]dz$$

$$=\sum_{r=1}^{q}\left[\sum_{s=1}^{n}\sum_{t=1}^{m} d_{is}\,(v)[\int_a^b d_{it}^{u-1}(z) r_{js}(z)dz] r_{rt}^{u-1}(k)\right]$$

Since $\int_a^b\left[d_{it}^{u-1}(z) r_{js}(z)\right]dz=c,\ c$ is constant. Then
$Q_{ij}^u(v,k)=c\sum_{r=1}^{q}[\sum_{s=1}^{n}\sum_{t=1}^{m}[\,d_{is}(v) r_{rt}^{u-1}(k)]]$ (11)
$Q_{ij}^u(v,k)$ is degenerate kernel.

we must prove that $Q_{ij}^n(v,k)$ is the degenerate kernel for $n=u+1$.

$$Q_{ij}^{u+1}(v,k)=\sum_{r=1}^{q}\int_a^b Q_{ij}\,(v,z)\,Q_{ir}^{u}(z,k)dz$$

$$=\sum_{r=1}^{q}\int_a^b\left[\sum_{s=1}^{n} d_{is}\,(v) r_{js}(z)\sum_{t=1}^{m} d_{it}^{u}\,(z) r_{rt}^{u}(k)\right]dz$$

$$=\sum_{r=1}^{q}\int_a^b\left[\sum_{s=1}^{n} d_{is}\,(v) r_{js}(z) d_{it}^1(z) r_{rt}^1(k)\sum_{t=1}^{m} d_{it}^{u-1}\,(z) r_{rt}^{u-1}(k)\right]dz$$

$$=\sum_{r=1}^{q}\int_a^b\left[\sum_{s=1}^{n}\sum_{t=1}^{m} d_{is}\,(v) d_{it}^1(z) d_{it}^{u-1}(z) r_{js}(z) r_{rt}^1(k) r_{rt}^{u-1}(k)\right]dz$$

Since these terms $r_{rt}^1(k) r_{rt}^{u-1}(k)$ and $d_{is}(v)$ are constants for the integral with respect to variable z, then we get
$Q_{ij}^{u+1}(v,k)=$
$\sum_{r=1}^{q}\sum_{s=1}^{n}\sum_{t=1}^{m}[d_{is}\,(v) r_{rt}^1(k) r_{rt}^{u-1}(k)\int_a^b\left[d_{it}^1(z) d_{it}^{u-1}(z) r_{js}(z)dz\right]]$

Since $\int_a^b\left[d_{it}^1(z) d_{it}^{u-1}(z) r_{js}(z)dz\right]=w,\ w$ is constant

$$Q(v,k)=w\sum_{r=1}^{q}\left[\sum_{s=1}^{n}\sum_{t=1}^{m} d_{is}\,(v) r_{rt}^1(k) r_{rt}^{u-1}(k)\right]$$

By using equation (11) and definition of the degenerate kernel then we obtain $Q_{ij}^{u+1}(v,k)$ is the degenerate kernel.
Therefore, the relationship holds $\forall, n\in$ N. such that N is the set of natural number.

2) Since $Q_{ij}(v,k)=\sum_{y=1}^{n} d_y\,(v) r_y(k)$ non-identically zeros for some subscript value of y. Then at least one term of $d_y(k) r_y(k)$ is nonequal to zero.
By mathematical induction we must show that $Q_{ij}^2(v,k),\ Q_{ij}^3(v,k),\dots,Q_{ij}^n(v,k)$ are non-identically zeros.
$Q_{ij}^1(v,k)=Q_{ij}(v,k)$, then $Q_{ij}^1(v,k)$is non-identically zero.
Assume that $Q_{ij}^n(v,k)$ non-identically zeros for $n=u$.

we must prove that $Q_{ij}^n(v,z)$ is non-identically zeros for $n=u+1$
then $Q_{ij}^{u+1}(v,k)=\sum_{j=1}^{q}\int_a^b Q_{ij}\,(v,z)\,Q_{ij}^u(z,k)dz$
since by supposing $Q_{ij}^u(v,k)$ and $Q_{ij}^1(v,k)$ are non-identically zeros $\forall, (v,k)\in G$.
Hence by theorem (4.2) the multiplication of them is non-identically zero.
Therefore, the relationship holds $\forall, n\in$ N.

Theorem 4.4
Assume the linear operator $W: C[a,b]\to C[a,b]$ is continuous on Banach space $Y(X)$
The function $h_i(v)$ is analytic over $[a,b]$ and $Q_{ij}(v,z)$ is continuous on
$G=\{(v,z): a\le v<z\le b\}$ such that,

$$\left\|Q_{ij}(v,z)p_i^m(z)-Q_{ij}(v,z)p_i^{m-1}(z)\right\|$$
$$\le L_i\left\|p_i^m-p_i^{m-1}\right\|\le L\left\|p_i^m-p_i^{m-1}\right\|$$

Such that $0<L_i<1$, and $L=\max\{L_i\}_{i=1}^{m}$ then W is contractive mapping.

Proof: From equation (9) we have

$p_i^m(v) = \sum_{j=1}^{q} \int_a^b Q_{ij}^m (v,k) h_j(k) dk, v \in [0, k_i], k_i < 1$ for any $n = m$, to show that W is contractive mapping. Let $p_i^m(v)$ and $p_i^{m-1}(v) \in C[a,b]$ by applying mathematical induction and the Euclidean norm $\|.\|$,

$$\left\|W\left(p_i^2(v)\right) - W\left(p_i^1(v)\right)\right\| = \left\|\sum_{j=1}^{q} \int_a^b Q_{ij}(v,z) p_i^1(z) dz - \sum_{j=1}^{q} \int_a^b Q_{ij}(v,z) p_i^0(z) dz\right\|$$

$$= \sum_{j=1}^{q} \int_a^b \|Q_{ij}(v,z) p_i^1(z) - Q_{ij}(v,z) p_i^0(z)\| \, dz$$

By using the Lipchitz condition $\|Q_{ij}(v,z) p_i^1(z) - Q_{ij}(v,z) p_i^0(z)\| \le L_i \|p_i^1 - p_i^0\|$, and $\sum_{j=1}^{q} f = qf$, we get

$$\left\|W\left(p_i^2(v)\right) - W\left(p_i^1(v)\right)\right\| \le \sum_{j=1}^{q} L_i \|p_i^1 - p_i^0\| \int_a^b dz = q(b-a) L_i \|p_i^1 - p_i^0\|$$

$$\left\|W\left(p_i^3(v)\right) - W\left(p_i^2(v)\right)\right\| = \left\|\sum_{j=1}^{q} \int_a^b Q_{ij}(v,z) p_i^2(z) dz - \sum_{j=1}^{q} \int_a^b Q_{ij}(v,z) p_i^1(z) dz\right\|$$

$$= \sum_{j=1}^{q} \int_a^b \|Q_{ij}(v,z) p_i^2(z) - Q_{ij}(v,z) p_i^1(z)\| \, dz$$

since $\|Q_{ij}(v,z) p_i^2(z) - Q_{ij}(v,z) p_i^1(z)\| \le L_i \|p_i^2 - p_i^1\|$, then

$$\left\|W\left(p_i^3(v)\right) - W\left(p_i^2(v)\right)\right\| \le \left|\sum_{j=1}^{q} L_i |p_i^2 - p_i^1| \int_a^b dz = q(b-a) L_i |p_i^2 - p_i^1| \right|$$

$$= q^2 (b-a)^2 L_i^2 \|p_i^1 - p_i^0\|$$

Therefore, in general $\left\|W\left(p_i^{n+1}(v)\right) - W\left(p_i^n(v)\right)\right\| \le q^n (b-a)^n L_i^n \|p_i^1 - p_i^0\|$

since $0 < L_i < 1$, then $0 < L_i^n < 1$ therefore $0 < q^n (b-a)^n L_i^n < 1$

Then W is contractive mapping.

ALGORITHM OF TECHNIQUE

Input: a, b, m, f, E

Step 1: Consider $p_i(v) = \sum_{j=1}^{n} \beta_{ij} p_i^j(v)$ be numerical solution of equation (1).

Step 2: Put $p_i^0(v) = h_i(v)$ for $i = 1,2,\dots,m$.

For $n = 1$: f

Step 3: Compute iterative kernel from equation (10).

Step 4: Find $\{p_i^1(v), p_i^2(v), \dots, p_i^n(v)\}$ in equation (9).

Step 5: Calculate $E_i^n = \left|\sum_{j=0}^{n} p_i^j(v) - p(v)\right|$.

if $E_i^n < E$ then go to output.

Step 6: Containing in this process to get the numerical solution.

End

Output: the results approximate results and E_i^n.

Note: E= error value.

5- EXAMPLES

Discussing the iterative kernel technique for solving the problem through numerical examples.

Example 1: Consider the system of integral equation

$$\frac{1}{2} v = \frac{1}{2} \int_0^1 v \; p_1(z) dz + \int_0^1 2v^2 \; p_2(z) dz$$

$$\frac{1}{4} v = \frac{1}{2} \int_0^1 v z^2 \; p_1(z) dz + \frac{3}{4} \int_0^1 vz \; p_2(z) dz$$

With exact solutions $p_1(v) = v$ and $p_2(v) = \frac{1}{2} v$.

Solution: By applying iterative kernel technique, obtains

$$p_1^0(v) = h_1(v) = \frac{1}{2} v, \; p_1^1(v) = \frac{1}{4} v, \; p_1^2(v) = \frac{1}{8} v \text{ and } p_1^3(v) = \frac{1}{16} v$$

$$p_2^0(v) = h_2(v) = \frac{1}{4} v, \; p_2^1(v) = \frac{1}{8} v, \; p_2^2(v) = \frac{1}{16} v \text{ and } p_2^3(v) = \frac{1}{32} v$$

Table 1: Comparison among numerical and exact results.

v	n	Exact solution of $p_1(v)$	Numerical solution by technique	Absolute error	Exact solution of $p_2(v)$	Numerical solution by technique	Absolute error
0.6	2	0.60000000	0.45000000	1.5000×10^{-1}	0.30000000	0.22500000	7.5000×10^{-2}
	4		0.56250000	3.7500×10^{-2}		0.28125000	2.8125×10^{-2}
	6		0.59531250	4.6875×10^{-3}		0.29765650	2.3437×10^{-3}
	8		0.59941406	5.8500×10^{-4}		0.29970703	2.9297×10^{-4}
	10		0.59992675	7.3750×10^{-5}		0.29996337	3.6621×10^{-5}
	12		0.59999084	9.1344×10^{-6}		0.29999542	4.5776×10^{-6}
	14		0.59999988	1.1444×10^{-7}		0.29999942	5.7220×10^{-7}
	16		0.59999995	5.9300×10^{-8}		0.29999997	7.1500×10^{-8}
	18		0.60000000	5.3450×10^{-9}		0.30000000	8.9000×10^{-9}
	20		0.60000000	2.6070×10^{-11}		0.30000000	1.1000×10^{-10}

Table 2: Comparison of least square error and running time.

		Computes for $p_1(v) = v$		Computes for $p_2(v) = \frac{1}{2}v$	
v	**n**	Least square error	Running time	least square error	Running time
0.6	2	643×10^{-3}	0:0:0.9632	2.5576×10^{-3}	0:0:0.9632
	4	223×10^{-7}	0:0:1.7864	4.3245×10^{-7}	0:0:1.7864
	6	287×10^{-8}	0:0:2.2432	6.7783×10^{-9}	0:0:2.2432
	8	672×10^{-12}	0:0:3.1168	7.9006×10^{-13}	0:0:3.1168

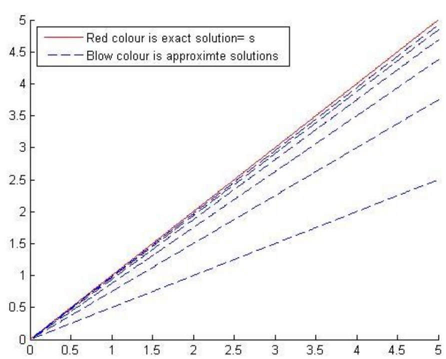

Figure 1: Describe numerical graphs and $p_1(v) = v$

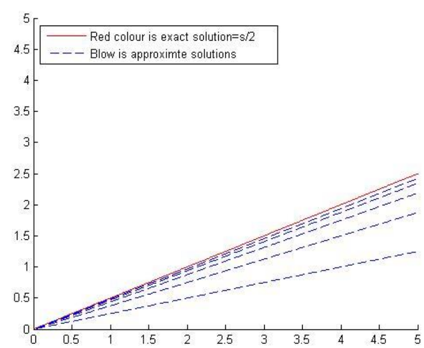

Figure 2: Describe numerical graphs and $p_2(v) = \frac{1}{2}v$.

Example 2: Consider the system of integral equation

$$\frac{1}{8}e^{v} = 3\int_0^1 z^3 e^{(v-z)} p_1(z)dz + \int_0^1 v e^{(v-z)}\ p_2(z)dz$$

$$\frac{1}{16}e^{v} = \frac{1}{4}\int_0^{13} 3v^5 e^{(v-z)} p_1(z)dz + \frac{1}{2}\int_0^1 v e^{(v-z)}\ p_2(z)dz$$

With exact solutions $p_1(z) = \frac{1}{4}e^v$ and $p_2(z) = \frac{1}{8}e^v$.

Solution: By applying iterative kernel technique, obtains

$$p_1^0(v) = h_1(v) = \frac{1}{8}e^v,\ p_1^1(v) = \frac{1}{16}e^v,\ p_1^2(v) = \frac{1}{32}e^v \ and\ p_1^3(v) = \frac{1}{64}e^v$$

$$p_2^0(v) = h_2(v) = \frac{1}{16}e^v,\ p_2^1(v) = \frac{1}{32}e^v,\ p_2^2(v) = \frac{1}{64}e^v \ and\ p_2^3(v) = \frac{1}{128}e^v$$

Table 3: comparison of numerical and exact results.

v	n	Exact solution of $p_1(v)$	Numerical solution by technique	Absolute error	Exact solution of $p_2(v)$	Numerical solution by technique	Absolute error
0.6	2	0.45552969	0.22776485	2.27764×10^{-1}	0.22776484	0.11388242	1.1388×10^{-1}
	4		0.42705909	2.8470×10^{-2}		0.21352954	1.4235×10^{-2}
	6		0.41197087	3.5588×10^{-3}		0.22598543	1.7794×10^{-3}
	8		0.45508484	4.4485×10^{-4}		0.22754242	2.2242×10^{-4}
	10		0.45547407	4.1194×10^{-5}		0.22773704	2.7803×10^{-5}
	12		0.45552274	6.9508×10^{-6}		0.22776137	3.4754×10^{-6}
	14		0.45552883	8.6880×10^{-7}		0.22776441	4.3440×10^{-7}
	16		0.45552959	1.8060×10^{-8}		0.22776479	5.4300×10^{-8}
	18		0.45552964	1.3500×10^{-9}		0.22776443	6.7800×10^{-9}
	20		0.45552969	1.600×10^{-10}		0.22776484	1.1042×10^{-9}

Table 4: Comparison of least square error and running time.

		Computes for $p_1(z) = \frac{1}{4}e^v$		Computes for $p_2(z) = \frac{1}{8}e^v$	
v	**n**	Least square error	Running time	Least square error	Running time
0.6	2	679×10^{-2}	:0.9504	543×10^{-2}	:0.9504
	4	762×10^{-5}	:1.6788	985×10^{-5}	:1.6788
	6	520×10^{-8}	:2.7740	452×10^{-7}	:2.7740
	8	651×10^{-11}	:3.0032	310×10^{-10}	:3.0032

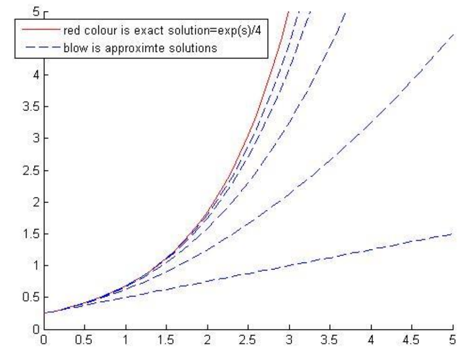

Figure 3: Describe numerical graphs and $p_1(z) = \frac{1}{4}e^{v}$.

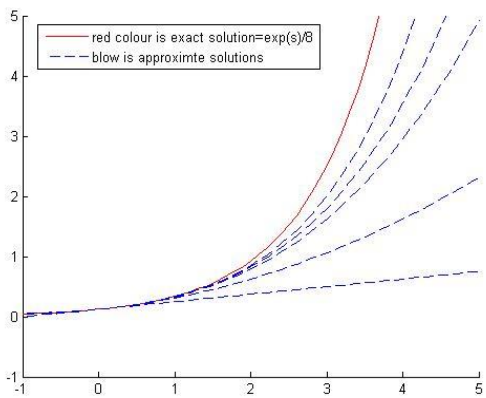

Figure 4: Describe numerical graphs and $p_2(z) = \frac{1}{8}e^{v}$.

6. CONCLUSION

In this paper, reformulating iterative kernel technique for finding an approximate solution for system Fredholm integral equation of the first kind. Examples were solved and good results are achieved by applying this technique. The algorithm in this study is efficient for solving this problem. Also, for the criterion of discussion, we computed the least square error and running time of the program, for supporting the efficacy of the technique applied on several examples and getting an approximate solution for the problems. Comparing the approximate solution with the corresponding exact solution results. Finally, concluding the following points:

1- Stated the new results.
2- Proved new theorems and formulas.
3- The iterative technique showed its applicability and effectiveness.

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